A Tentative Model for Generic Catastrophic Systems

Katz School of Science and Health

D. Tony Sun (dsun1@mail.yu.edu), Mathematics/MA Student

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ABSTRACT Making a sketch for the example of tornadoes' modeling, this presentation lays out a brief modeling approach by dealing with objects of interest along-side with the physical environment, considering several scalar fields like the temperature.

INTRODUCTION When a complex social, catastrophic problem is presented with various outlets of pressing, practical problems to be address, an analytical mathematician may start with gaining the *access* to the facets of it, since its entirety is hardly predictable henceforth transparency unguaranteed. My practical interest in this potential project sprung from the ongoing need for residents along the east and west coasts of United States to evacuate from typhoons/hurricanes (differentiated from forest fires) and possibly tsunamis, such as the Hurricane Katrina which took place in Louisiana in 2005.

Generically, a "system" of a catastrophe consists of the *objects* (e.g., resident civilians during a flood), and then there is the *environment* also in terms of an in-situ analysis, in which time is a dominant variable. Here take tornadoes (a.k.a. twisters, with minimum wind speed above 65 miles per hour) as example; the rationale for modeling a tornado object along-side with the wind field is because it has much strong wind speed/flow there, hardly negligible. Hence my attempt here is identifying a tornado curve, say C, parametrized as

$$\mathbf{r}(t) = \begin{bmatrix} e^{-t} \cos(100t) \\ e^{-t} \sin(100t) \\ e^{-t/100} \sin(t) \end{bmatrix}$$

where time t can start from zero to infinity–note that if (normally) the system also starts simultaneously at $t = 0$, this assumes the twister begins at the bottom and "generates" itself onward, rather than a pre-existing object prior to the start of the system.

SUGGESTED MODEL(S) OF ENVIRONMENT/DYNAMICS

Under the Cartesian *xyz*-coordinates, suppose z aligns "vertically" with the altitude. I consider three scalar fields for the environment *before* modeling the dynamics, as follows:

- density of air in space $\rho(x, y, z)$
- temperature field $T(x, y, z)$ as the intensity of heat;
- air pressure field as a function of temperature and altitude

$$P(x, y, z) = f_P\Big(T(x, y, z), \quad z\Big)$$

the population of the former is normally by run-time, empirical data. Then the acceleration would be the effects of the pressure gradient

$$\vec{a} \;=\; -\frac{1}{\rho}\,\nabla P$$

which would alternate/drag the twister at any given time t.

Alternatively, an approximation without temperature, etc. is a model for weather systems call the *Lorenz Equations* (Lorenz, 1963) that models the location-coordinates by their derivatives on time, i.e., where it tends to go

$$\frac{dx}{dt} = \sigma(y - x)$$

with x, y, z mutually co-dependent on each other, where sigma is one of the empirical "classical parameters."

CONCLUSION Behind this sketch is my continued study on *catastrophe theory* as a research topic of research coined in late 1970s which would fall under dynamical systems to-day. A more detailed modeling would be my future project.

REFERENCES

Lorenz, E. N. (1963). Deterministic Nonperiodic Flow. *Journal of Atmospheric Sciences, 20*(2), 130-141.